Symmetry Analysis of Stucco Ornaments from the Great Seljuk Period

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Abstract

Symmetry is the basic feature of surfaces that are decorated with periodic arrangements of the same shape in two dimensions and are lined up harmoniously and visually without gaps. Within the framework of mathematical group theory, there are precisely 17 different arrangements classified as "wallpaper groups". Each of these compositions has unique characteristics that reflect the skill and creative vision of the artisans responsible for their creation. The study of these patterns in different civilisations provides practical insights into cultural exchange and its impact on the formation of artistic practices between various groups. Using the concept of wallpaper group, this study analyses the plaster decorations on the interior walls of several mosques built during the Seljuk period in Iran, spanning the 11th and 12th centuries. The motifs used in these buildings have been strongly influenced by the earlier artistic traditions of Iran, especially those of the Sassanid period. It is worth mentioning that the school of Iranian artisans demonstrated remarkable continuity in style and technique during the four centuries of Arab rule after the Islamic conquest in 651 AD. This enduring heritage reflects the stability and long-term influence of Iranian art, which has managed to retain its distinctive personality despite significant cultural changes. By examining the stucco decorations, this study aims to contribute to a deeper understanding of the mutual influence of artistic traditions and cultural identity during a pivotal period in Iranian history.

Keywords: Great Seljuk Period, Stucco Ornaments, Symmetry, Wallpaper Groups, Correlation, Principal Components Analysis

1. Introduction

The relationship between geometry, mathematics, and Islamic architectural decoration has long been the subject of research interest, highlighting the deep connection between abstract mathematical principles and Islamic artistic traditions. Islamic architecture is known for its intricate decorative patterns, characterised by extensive use of geometric motifs that convey both visual beauty and spiritual symbolism. Found in plasterwork, tiling, woodcarving, and metalwork, these geometric shapes demonstrate not only artistic creativity but also represent an advanced understanding of mathematics, particularly geometry.

The study of symmetry in art and architecture provides a profound understanding of the cultural and mathematical perspectives of past civilisations. In the field of Islamic art, especially during the period of the Great Seljuks (11th to 12th century AD), plaster decorations are one of the important effects of symmetry that can be studied using group theory.

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Symmetry, the unbroken repetition of identical forms, is considered a fundamental aspect of geometric design in Islamic architecture [1]. The wallpaper group represents the artistic and mathematical complexity hidden in the handiwork of artists who have used these patterns in architectural decorations [2].

Critchlow believes that Islamic art is full of symbolism. By careful use of geometry, Islamic artists created patterns that could theoretically be expanded infinitely [3]. Gülrü Necipoğlu, in the book "Top-kapı Scroll: Geometry and Decoration in Islamic Architecture", examines the practical applications of geometry in the design process of Islamic architecture. Necipoglu provides a detailed analysis of the 15th-century Topkapı Scroll, a key document for understanding the methods used by Islamic artisans to design and execute complex geometric motifs. Her research reveals that these patterns were not merely decorative but were formed through precise mathematical calculations and grid systems that enabled their repetition in various architectural media [4]. Similarly, the book "Islamic Geometric Patterns" by Eric Broug (2008) provides a comprehensive study of the methods used to create geometric patterns in Islamic art. Broug shows how Islamic architects and artists employed mathematical principles such as symmetry, proportion, and repetition to create complex designs. His work has played a crucial role in revealing the mathematical accuracy hidden within these complex designs [5].

This article analyses the plaster decorations on the interior walls of mosques built during the Great Seljuk period, focusing exclusively on their use of symmetry, which is influenced by earlier Iranian artistic traditions, particularly the Sassanid period. The continuity of these Iranian decorative techniques, despite four centuries of Arab rule after the Islamic conquest in 651 AD, is a testimony to the stability and versatility of Iranian artistic traditions. The results contribute to a deeper understanding of the mathematical foundations of Islamic art and underscore the significance of cultural continuity in the development of architectural decoration throughout the centuries.

2. Stucco Ornamentation in Great Seljuk Architecture: Sources of Inspiration

The Great Seljuk period (11th to 12th century AD) was a turning point in the history of Islamic art and architecture, marked by the emergence of new decorative styles that had a profound impact on Islamic artistic traditions. One of the most important types of decorative arts of this period was plastering, which was used to decorate important buildings such as mosques. The rich visual language of these stuccoes can be traced to several cultural and historical influences, each uniquely incorporated into Seljuk art. The Seljuks inherited a rich cultural heritage and skillfully combined traditional motifs with Islamic themes, transforming them into abstract and non-visual forms that adhered to Islamic artistic principles. This combination is especially evident in the adaptation of floral motifs, such as Islimi, in the plaster decorations of the Seljuk period. These motifs demonstrate a clear continuation of the Sassanid artistic heritage. However, they have been recreated in a manner consistent with Islam's tendency towards abstraction and its conscious avoidance of representing human figures.

One of the primary sources of inspiration for Seljuk decorative arts was the rich heritage of pre-Islamic Iranian art, especially the Sassanid tradition. The Sassanid Empire (224-651 AD) developed a sophisticated aesthetic that integrated vegetal, geometric, and pictorial motifs into its architecture and decorative arts, particularly through stucco reliefs [6]. The Sassanid stucco, of which outstanding examples are found in the Ktesifon/Ctesiphon Palace or Tak-ı Kisra, featured complex motifs that included ivy, flowers, geometric patterns, animal motifs, and mythological creatures, which were later reflected in Seljuk stucco. Another important source of inspiration for Seljuk art is the artistic heritage of the Ghaznavid period, which draws heavily on the traditions of the Sassanid period. The Ghaznavids, who ruled parts of Iran, Central Asia, and the Indian subcontinent from the 10th to the 12th century, served as a cultural bridge between the ancient Sassanid traditions and the Islamic aesthetics that developed later in the Seljuk period. Sassanid influences, characterised by intricate plant motifs, artistic plasterwork, and symbolic representations, strongly shaped Ghaznavid art, as seen in their architectural decorations [7]-[13].

3. Wallpaper Group

In two dimensions (2 DIM), identical particles can be arranged in 17 different ways to cover the surface periodically. This knowledge, derived from group theory, was a milestone in crystallography. In 1924, Pólya postulated that the 2 DIM artwork, which can be found as decoration in many buildings, also obeys group theory [14]. Mu¨ller tested P´olya's idea by looking at the artwork in the Alhambra Palace in Granada in 1944 [15]. The wallpaper group includes one-, two-, three-, four-, and sixfold rotational symmetries, in addition to reflections and translations [16]. According to the International Union of Crystallography, the group notations are p1, p1m1, p1g1, c1m1 with rotational symmetries of n = 1, p211, p2mm, p2mg, p2gg, c2mm with rotational symmetries of n = 2, p3, p3m1, p31m with rotational symmetries of n = 3, p4, p4mm, p4gm with rotational symmetries of n = 4 and p6, p6mm with rotational symmetries of n = 6, where p stands for a primitive cell, c contains an element in the center of the cell. m is a mirror symmetry, g is a glide reflection resulting from the combination of m with the translation. In the case of an ornament, the group to which it belongs is determined by the combination of these symmetry properties.

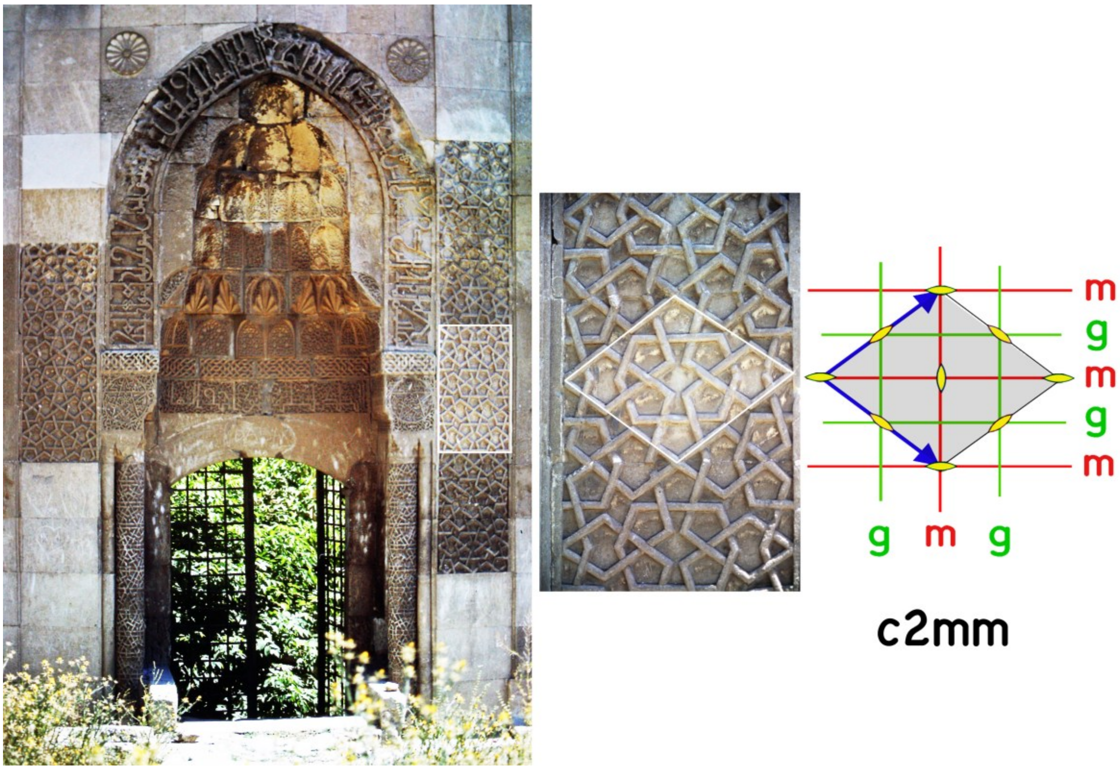

Figure 1: The mausoleum of Mama Hatun as an example of Rum Seljuk architecture in Tercan. On the right are the symmetry elements for the unit cell, which is highlighted in the ornament in the middle (from Ref [17]).

3.1. Wallpaper in Rum Seljuk

To substantiate this procedure, we present in Figure 1 a photograph of the gate of the Mama Hatun Tu¨rbesi in Tercan, Erzincan, the mausoleum of Mama Hatun (13th century). This is an outstanding example of the Rum Seljuks in Eastern Anatolia. A rectangular area on the portal with a particular ornament is highlighted and shown again in the centre, with a smaller diamond shape also highlighted, representing the unit cell (repeat unit) of the ornament. The symmetry elements of the unit cell are shown on the right. We discern the translation vectors (blue arrows) that span the unit cell, the axes of twofold rotations (yellow), the mirror reflection lines m (red) and the glide reflections g (green). These details of the unit cell are typical for the wallpaper group c2mm [17].

3.2. Wallpaper as FingerPrint

In later years, it was found that a particular cultural group produces certain selected symmetries [18]. In other words, the artisans of a cultural group tend to create their own characteristic patterns. We have adopted this insight and analysed the symmetries of different cultural groups to find similarities in their artworks and thus relationships between the groups. To achieve this goal, we determine the wallpaper group of an artwork under consideration and repeat this process for all the artefacts at hand to obtain a distribution of the occurrence of symmetries. This procedure is then applied to other civilisations. For example, we found that fourfold symmetric artefacts dominate Armenian and Eastern Roman artwork, while Arab Muslims have additional two- and sixfold symmetrical artworks. Such an analysis allows us to recognise cultural groups with similar habits. In fact, we can identify two distinct clusters, which we call the ”Muslim” group, composed of the Arabs, the Seljuk Empire and the Seljuks of Rum, and the ”Christian” group, composed of the Hellenistic colonies in Western Anatolia, the Eastern Roman Empire (Byzantium), the Armenians and Andalusia [19]. Persian arts and crafts during the classical period, as well as the Middle Ages, cannot be categorised in either group and form a school of their own [20].

Figure 2: On the left, a general view of the brick-covered inner surfaces of the dome of the Friday Mosque in the city of Ardistan, Isfahan Province, is shown. The depiction of the ornament is in the centre (from Ref. [7], Photographs 361 and 372). The shaded square area is the unit cell of the pattern, which is defined by the translation vectors (blue) on the right side. Here, we display the symmetry elements typical of the group p4mm.

4. Seljuk Stucco Decorations

The stucco ornamentation of the Seljuk Empire has been the subject of extensive research [7]-[13]. In the following, we will examine the wallpaper group of Seljuk stucco works, which are mainly found on the interior walls of their mosques. Here, we will give a few examples to illustrate how we analyse the artwork and the data that form the basis of the mathematical analysis.

Figure 2 shows, on the left, a general view of the brick-covered inner surfaces of the dome of the Friday Mosque in the city of Ardistan, Isfahan Province. An area on the decorated dom marked with white borders is enlarged in the centre of the figure. The dark-highlighted area is the unit cell of the pattern, which repeats itself to form the entire decoration. The symmetry properties are shown on the right. Blue arrows are the translation vectors of the pattern and define the unit cell. Red lines are the mirror reflections of the pattern, while green lines indicate the glide-reflection lines. The small red and green squares in the pattern are the fourfold-rotation axes perpendicular to the plane of the paper. The yellow diamonds are twofold-rotation axes. All these symmetry properties show that the ornament shown here belongs to the symmetry group p4mm.

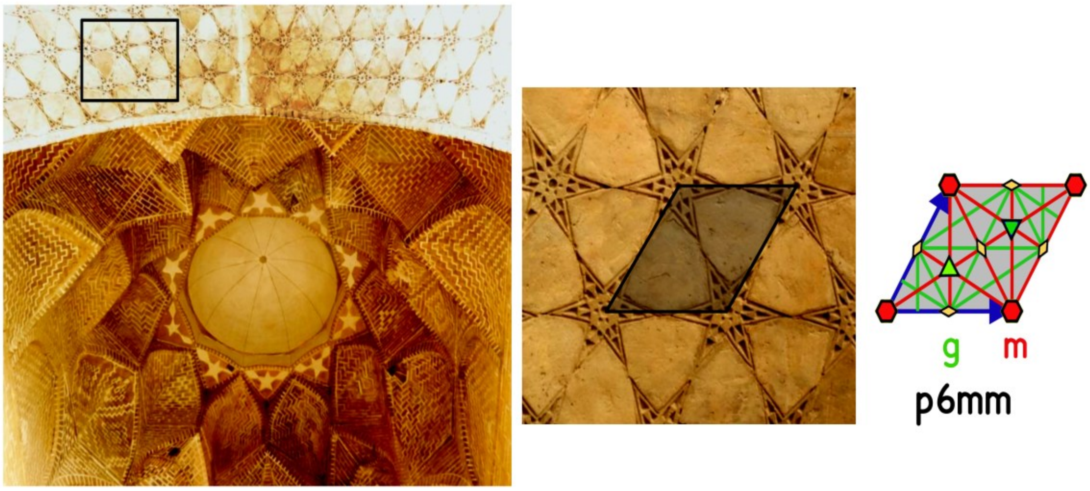

Figure 3: Figure 3: On the left, we see a general view of the interior surfaces of the dome of the Friday Mosque in Sin, where an area on its border, decorated with *brick-end plug decorations*, is marked. This area contains the pattern of interest and is shown in the centre (from Ref. [7], Photographs 275 and 276). The shaded rhombic area is the unit cell of the pattern. Its symmetry elements are shown on the right side. The translation vectors (blue) span the unit cell, which is the repeat unit, typical for the group p6mm.

Another example of the stucco decorations is presented in figure 3. This figure shows the brick-end plug decorations that form the interior surfaces of the dome of the Friday Mosque in the city of Sin, Isfahan Province. On the left is depicted the dome of the mosque with its border decoration, a portion of which is marked and enlarged in the centre of the figure. It is sixfold symmetric. Its unit cell is indicated and highlighted in dark. The unit cell is spanned by the translation vectors (blue lines) separated by 60˚. It contains, besides the 60˚ rotations (red hexagons), two different types of threefold- (green triangles) and three different types of twofold-rotation (yellow diamonds) axes perpendicular to the plane of the paper. There are mirror-reflection lines running at 60˚ intervals through the sixfold-symmetry centres. The threefold symmetry axes are located at midpoints of each pair of sixfold symmetry centres. There are six glide reflections at every 60˚, which run through twofold-symmetry axes. The symmetry group is hexagonal p6mm.

We have observed two typical ornaments with rather obvious overall symmetries, namely fourfold (Fig. 2) and sixfold (Fig. 3). We now present another example with a relatively rich unit cell, but a simpler overall structure. On the left of figure 4. We present a photograph of the interior of the southern iwan of the Friday Mosque in Ardistan, Isfahan Province, showcasing a wall stucco pattern highlighted by a white-bordered square. This area is shown at the centre of the figure, where the unit cell of the pattern is darkly highlighted. On the right, we have the unit cell spanned by the translation vectors (blue) and thin black lines. The mirror- and glide-reflection axes (red and green lines, respectively) repeat themselves periodically parallel to each other and have the same distance to adjacent lines. This group is c1m1 and contains no rotations, except for the trivial 360˚ rotation.

Figure 4: On the left, we see a general view of the interior of the southern iwan of the Friday Mosque in Ardistan, Isfahan Province, where an area of wall stucco decoration is marked. This area contains the pattern of interest and is shown in the centre (from Ref. [7], Photographs 338 and 341). The shaded area is the unit cell of the pattern. Its symmetry elements are shown on the right side. The translation vectors (blue) span the unit cell, which is typical for the group c1m1 with its mirror and glide symmetry.

There are also stucco decorations having intriguing symmetry properties with rather elaborate repeat units. Figure 5 presents a prominent example of such a pattern from the mihrab of the Melik Mosque in Kirman. On the left side, we see the photograph of the ornament. The graphic in the centre represents the artwork, with the shaded area indicating the unit cell. On the right, we present the symmetry features, which include five fourfold axes of rotation perpendicular to the plane of the unit cell as well as four twofold rotation axes of symmetry (yellow). Together with the mirror lines (red) and the glide lines (green), this results in the wallpaper group of the artwork as p4gm.

Figure 5: On the left, we see the wall decoration on the side of the mihrab of the Melik Mosque in Kirman. The depiction of the ornament is centred (from Ref. [7], Figure 175). The shaded square area is the unit cell of the pattern, which is defined by the translation vectors (blue). On the right side are the symmetry elements, which are typical for the group p4gm.

We selected 86 stucco ornaments from Sokhanpardaz's dissertation [7] and examined them for their symmetry properties, as illustrated in a few cases in Figs. 1-5. We then compiled the wallpaper groups determined in this way for all available ornaments. The results are presented in Table 1, which shows the frequency of occurrence of a particular symmetry, both in numbers (#) and per cent (%). It is evident and interesting to note that the artisans particularly favour the ornaments c2mm, p4mm, and p6mm, which are striking due to their multiple mirror reflections. This choice is a characteristic feature of Seljuk stucco decoration.

p1	**p1m1**	**p1g1**	**c1m1**	**p211**	**p2mm**	**p2mg**	**p2gg**	**c2mm**
6 / 7.0	1 / 1.2	2 / 2.3	6 / 7.0	0 / 0.0	2 / 2.3	2 / 2.3	1 / 1.2	20 / 23.3
p3	**p3m1**	**p31m**	**p4**	**p4mm**	**p4gm**	**p6**	**p6mm**	**# / %**
0 / 0.0	2 / 2.3	2 / 2.3	1 / 1.2	11 / 12.8	2 / 2.3	8 / 9.3	20 / 23.3	86 / 100.0

Table 1: The distribution of symmetry in the Seljuk stucco decorations. We analysed 86 artefacts and divided them into 17 wallpaper groups. The occurrences, listed in numbers (#) and per cent (%), are shown in the table.

5. Comparison of Medieval Work

It is of great interest to compare the choice of symmetry groups of the craftsmen who created the stucco decorations in the interiors of Seljuk mosques with what we know about other medieval civilisations. We have previously studied several groups and present the complete data in Table 2. It is a tedious task to compare the bare numbers of the individual cultural groups listed in Table 2 and come to an acceptable conclusion. Therefore, we resort to statistical tools based on mathematics. In the following, we utilise principal component analysis (PCA), a technique for reducing dimensions while retaining essential information. Table 2 shows the habits of cultures in 17 components, and PCA reduces them to two.

6. PCA on Stucco Work

In PCA, the goal is to map the data into 2DIM to recognise the trends inherent in the observations. In other words, the data is transformed into a new coordinate system so that the most important information in the data can be easily recognised. The new dimensions are the principal components that represent the largest variation [22]. They are uncorrelated variables that are linear combinations of the original. We used the occurrences in Table 2 and calculated the correlation of artefacts in nine cases, utilising the programming language R. The results are shown in Figure 6.

	Hellen	E Roman	Armen	Andalus	Arabs	R Seljuk	Persia	Seljuk E	SDSE
p1	9.8	14.0	0.8	1.0	0.0	0.3	4.2	0.8	7.0
p1m1	1.6	9.7	4.9	0.0	0.0	0.0	3.7	0.0	1.2
p1g1	0.0	0.9	0.0	0.0	0.0	0.0	0.0	0.8	2.3
c1m1	1.6	4.4	2.4	0.0	0.0	0.6	10.3	0.0	7.0
p211	0.0	2.6	0.0	1.0	3.6	1.1	1.9	2.4	0.0
p2mm	6.6	9.7	5.7	1.9	5.8	8.8	1.4	6.4	2.3
p2mg	1.6	5.3	0.8	0.0	0.4	0.8	6.5	0.8	2.3
p2gg	0.0	0.9	0.8	0.0	0.0	0.6	4.2	3.2	1.2
c2mm	6.6	3.5	1.6	1.9	12.9	8.5	29.4	11.2	23.3
p3	0.0	0.0	0.8	1.0	0.0	0.8	0.9	0.0	0.0
p3m1	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	2.3
p31m	0.0	0.0	2.4	0.0	0.4	1.9	0.5	1.6	2.3
p4	1.6	1.8	12.2	27.2	4.4	8.2	5.1	9.6	1.2
p4mm	62.4	35.1	54.5	51.5	28.9	27.8	15.0	25.6	12.8
p4gm	6.6	11.4	4.9	5.8	5.3	5.2	0.9	12.8	2.3
p6	0.0	0.9	1.6	2.9	4.4	8.0	3.8	8.0	9.3
p6mm	1.6	0.0	6.6	5.8	33.8	27.5	11.7	16.8	23.3

Table 2: Occurrence of the individual symmetry groups (in per cent) in the artefacts of eight cultures. "Hellen" stands for Hellenistic, "E Roman" for the Eastern Roman Empire, also known as Byzantium, "Armen" for the Armenians, "Arabs" for the group from the Arabian Peninsula, "Andalus" for the Moorish kingdom in Andalusia, "Seljuk E" for the Seljuk Empire in Persia [1, 21], "R Seljuk" for the Rum Seljuks in Asia Minor and "SDSE" for the stucco decoration in the Seljuk Empire. The rotational symmetries are summarised in groups. The data in the first eight columns have already been reported in an earlier article [20].

In the PCA analysis, the civilisations appear in three groups. The Arabs, the Rum Seljuks and the Seljuk Empire form a cluster, as do the Armenians, the Hellenistic colonies and Andalusia. The Eastern Roman Empire is close to this last group. We can say that the Arabs, the Rum Seljuks and the Seljuk Empire created works of art with similar symmetry; the Armenians, the Hellenistic Colonies, the Moors in Andalusia, and the Eastern Romans created similarly related works of art. The Persian artisans developed their own distinctive style, which is also evident in the stucco decorations of the Seljuk Empire (SDSE).

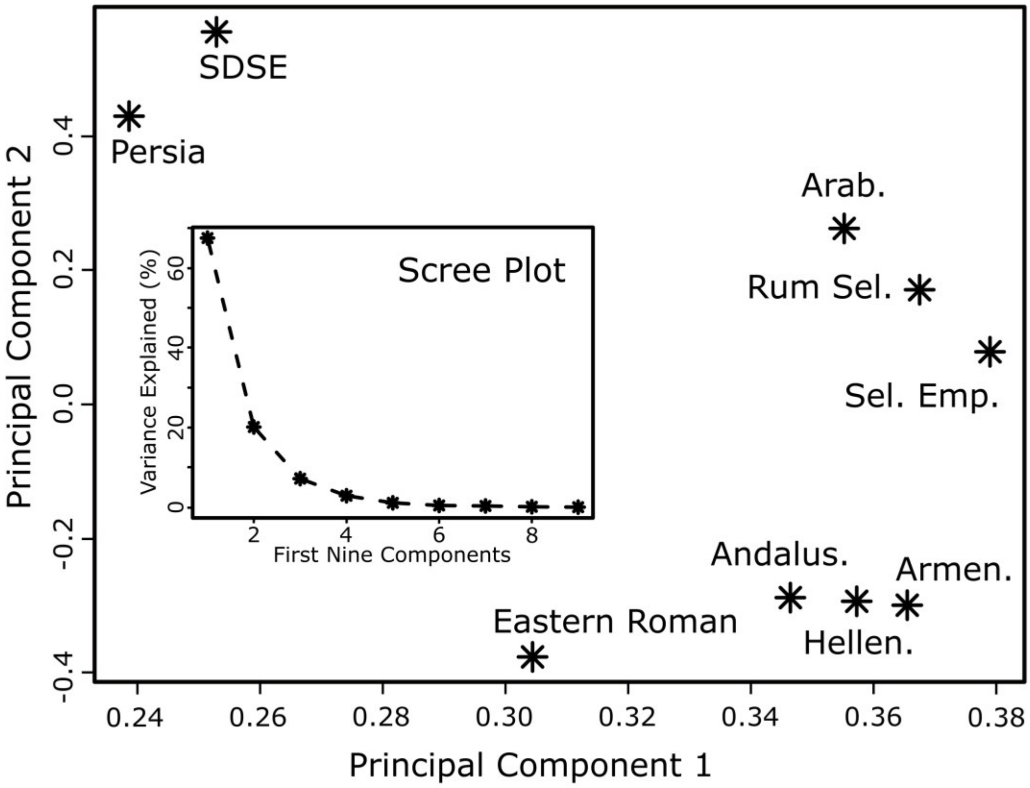

Figure 6: The result of the PCA analysis. The data from Table 2 are reduced into two main components, allowing the presentation of information about the symmetry-related habits of different civilisations in a 2DIM plot. The inset is a scree plot that reflects the degree of reliability of the PCA analysis.

The process of PCA is an approximation that helps us to distinguish some latent features in the data. Its reliability can be tested using the scree plot, which is a calculation of the variances in the data for each principal component under consideration [23]. The scree plot for the present PCA calculations is shown as an inset in the figure. It shows that the first component of the PCA representation carries about 68% of the information, and the second component about 20%. The results presented in Figure 6 and the subsequent conclusions are accurate with a reliability of 88%.

To corroborate the PCA results, Figure 7 displays the habits of the different cultural groups as histograms, using 17 plane symmetries that correspond to the wallpaper group. In organising the figure, we have used the PCA results from Figure 6 to distinguish three clusters. On the left are the Hellenistic colonies, the Armenians, the Eastern Roman Empire, and Andalusia, forming a single cluster. The second distinct cluster in the middle consists of the Arabs, the Seljuk Empire, and the Seljuks of Rum. Here, we consider the brick-laying artwork of the Seljuk Empire, primarily found in mosques, minarets, caravanserais, and madrasas. Finally, on the right-hand side, the art of the Persian school is on display.

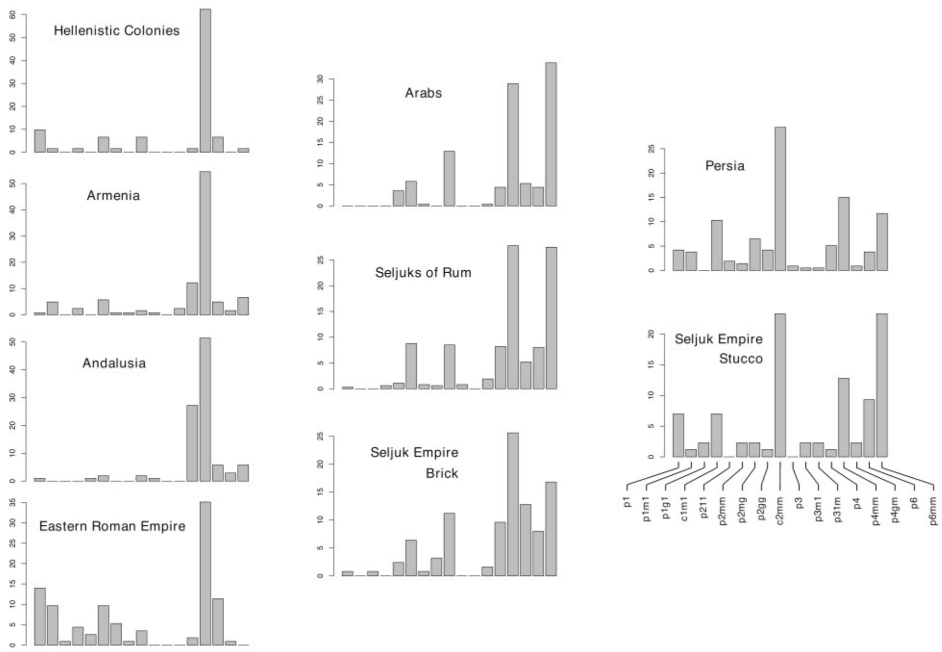

Figure 7: The data from Table 2 are plotted in three groups of civilisations. In each plot, the abscissa represents the wallpaper groups, and the ordinate their percentage of occurrence. The grouping is taken from the PCA results. Seljuk Empire/Brick is a compilation of results gathered from various existing sources (see text) regarding the buildings in Iran. At the same time, Seljuk/Stucco refers to the collection of symmetries found in the interiors of mosques.

Figure 7 vividly shows that each symmetry distribution has its peculiarities, and more importantly, that the similarities and differences between civilisations are directly recognisable. We can see that the fourfold symmetry predominates in the first group on the left. Other symmetries are present but of lesser importance. We can also see that the Eastern Roman Empire has an additional weight in one- and two-fold symmetry. We observe in Figure 6 that the value of the principal component PC1 is significantly smaller for this group than for the other three.

The middle part of Figure 7 shows the symmetry distributions observed among the Arabs, in the brick-laying of the Seljuk Empire and among the Seljuks of Rum. Here, the spectra are dominated by fourfold and sixfold rotational symmetries with some twofold contributions.

On the right, we have the Persian arts and crafts, along with the symmetrical distribution of stucco decorations in the monuments of the Seljuk Empire. The double-mirror contributions c2mm, p4mm and p6mm dominate the distribution with some contribution of one- and twofold symmetry.

Figure 7 is essentially a visualisation of Table 2, which presents the data as histograms and arranges them in the correct order.

7. Results

This study demonstrates how mathematical concepts—particularly the classification of two-dimensional symmetries through wallpaper groups—can serve as analytical tools to decode the geometric logic underlying medieval Iranian ornamentation, thereby revealing the shared structural principles between artistic creativity and mathematical thought. By applying the mathematical framework of wallpaper groups to Seljuk-period stucco ornaments, the research highlights how principles of symmetry and periodicity—central to mathematical thought—were intuitively and aesthetically employed by historical artisans.

This enriches the understanding of Islamic geometric design from a mathematical perspective and demonstrates the universality of mathematical order as a foundation of artistic expression.

The stucco decoration of the Great Seljuk period is an example of a complex amalgamation of diverse artistic influences, including pre-Islamic Iranian heritage. Rather than simply repeating these sources, the Seljuqs creatively reinterpreted these influences and skillfully reconciled them with the spiritual and aesthetic ideals prevalent in the Islamic world they ruled. The artistic productions of the Seljuk Empire have been the subject of numerous scholarly studies, which have revealed significant artistic achievements reflected in its architecture and decorative elements. A visual examination of Seljuk stucco reveals similarities with Iranian art, highlighting the continuity of artistic traditions across time and geography. To facilitate a comprehensive analysis, we have compiled a collection of 125 Seljuk designs extracted from various architectural structures, such as mosques and minarets. These motifs show the distinctive technique of Seljuk art, which is evident not only in the interior but especially in the exterior of these structures. The results of our analysis are presented in Table 2 under the title "Seljuq Empire".

A comparison of these values with those of other civilisations, using principal component analysis (PCA), reveals an important connection with the Roman and Seljuk Arab art traditions. On the other hand, an examination of the symmetry features in stucco decorations, as shown by SDSE in the last column of Table 2, shows a strong reliance on Iranian artistic practices, as shown in Figure 4. This detailed understanding of the decorative stucco of the Seljuk era not only accentuates their creative significance but also reveals the complexity of the cultural interactions that contributed to its development during this pivotal historical period.

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